A Hybrid Dynamic QRDM and ZF Detection Algorithm for MIMO-OFDM Systems

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***Abstract*—An effective signal detection algorithm with low complexity is presented for MIMO-OFDM systems. The proposed technique combines a dynamic QR decomposition based M-Algorithm (QRDM) and Zero-Forcing (ZF) detection. In the systems, QRDM is executed for the first $N_T-\lceil N_T/3\rceil+1$ layers, ZF detection is used for the last $\lceil N_T/3\rceil-1$ layers, where N_T means the number of transmitter antennas. Compared with the conventional QRDM algorithm, this approach is simple and has low complexity because not all branches are searched. Furthermore, the performance of this algorithm is similar to that of the conventional QRDM algorithm. The receiver detection simulation is based on five paths Rayleigh fading environment without channel coding, assuming the channel matrix H is perfectly known by the detector. From simulation results, the complexity of QRDM-ZF detection algorithm for 4 transmitter antennas and 4 receiver antennas systems with quadra-binary-shift-keying (QBSK) modulation is reduced by 29.41% on average and it is reduced by 30.08% on average for 16-quadra-amplitude-modulation (QAM). The performance degradation is about 2dB at BER=10^{-3}.The proposed hybrid QRDM-ZF detection can be used for a MIMO-OFDM receiver requiring not very high performance but needing very low complexity.**

Keywords-MIMO-OFDM;QR-decomposition;Zero-Forcing; QRDM algorithm; QRDM-ZF algorithm .

I. INTRODUCTION

The multiple-input multiple- output (MIMO) architecture provides significant capacity gain in wireless channels. And orthogonal frequency division multiple (OFDM) has become a popular technique for transmission of high data rate wireless communication because of its inherent error susceptibility in a multipath environment. OFDM based on MIMO is a key technique for the next generation wireless communication. In the MIMO-OFDM system, a signal detection algorithm with a low complexity and high BER performance is a troublesome subject. Therefore, lots of signal detection algorithms have been proposed [1]-[4]. There are QR [5], Zero-Forcing (ZF) [6] and a dynamic QR decomposition based M-Algorithm (QRDM) [7]. Among those detection algorithms, the QRDM algorithm has very high performance. Unfortunately, since this algorithm is a tree search process, the detection complexity will be highly increased when the number M increases.

In this paper, a simple and efficient detection technique based on hybrid QRDM and ZF is presented for MIMO-OFDM systems, for the first time. A new parameter $\lceil N_T/3\rceil$ has been adopted, where N_T means the number of transmitter antennas. In the first part of $N_T-\lceil N_T/3\rceil+1$ layers, QRDM algorithm is executed. Then ZF detection is used for the second part including last $\lceil N_T/3\rceil-1$ layers. Before detection, the channel matrix H has been sorted as the column norm from the minimum to the maximum. By this way, the performance of the first detected signals has enhanced greatly comparing with the conventional QRDM algorithm. The reason why ZF detection is used is that the performance of the first part detected layers is the best in all layers. When the first signals are reserved, its interference to other layers may be cancelled, and then the ZF algorithm will be much more reliable in the second part detection. Form simulation results, compared with QRDM algorithm, the QRDM-ZF detection performance is decreased 2dB at BER=10^{-3}. However, its complexity has decreased significantly.

The rest of this paper is organized as follows. The MIMO-OFDM systems model is described In Section II. In Section III, several conventional MIMO detection algorithms are briefly introduced. In section IV, a low complexity hybrid QRDM and linear ZF detection is proposed. The simulation results are presented in section V and Section VI is conclusions.

II. SYSYEM MODEL

A. MIMO-OFDM Systems

Consider a communication system with N_T transmitter antennas and N_R receiver antennas ($N_R \geqslant N_T$), denoted by (N_T,N_R). Fig.1 (a) is the structure of a MIMO-OFDM transmitter. At the transmitter the input information is de-multiplexed and coded to generate N_T symbol streams by serial-parallel converting. The symbol streams are lunched into the constellation mapping, the Inverse Fast Fourier Transform (IFFT) modulators and added cyclic prefix (CP). Finally, the number of N_T OFDM signals is transmitted by every transmitter antenna. Fig. 1(b) shows the block structure of a MIMO-OFDM receiver. Each receiver antenna receives signals from all transmit antennas.

After the cyclic prefix is removed, every received signal passes through a Fast Fourier Transform (FTT) block for demodulation. The receiver signal after demodulation, at receiver antennas can be given by

$$y = Hx + n \tag{1}$$

Where H is the complex-valued $N_R \times N_T$ matrix of quasi-static flat Rayleigh fading channel, y is a $N_R\times1$ vector of received signals, x is the transmitted signals with $N_T\times1$ vector, and n is $N_R\times1$ additive noise white Gaussian (ANWG) vector. Let's denote $y=[y_1,...,y_{N_R}]^T$, $x=[x_1,...,x_{N_T}]^T$ $n=[n_1,...,n_{N_R}]^T$, and

$$H=(h_{i,j})_{(1\leqslant i\leqslant NR\ ,\ 1\leqslant j\leqslant NT)} \tag{2}$$

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Where $h_{i,j}$ is a fading channel impulse response from transmitter antenna *j* to receiver antenna *i*.

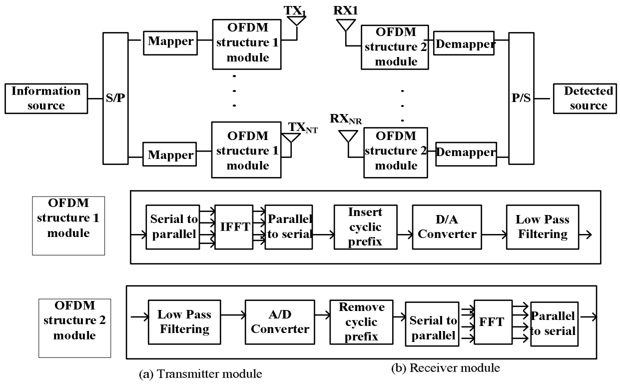


Figure. 1 The structure of MIMO-OFDM system

B. Model Assumptions

Additionally, in order to detect transmit signals, we make some assumptions:

(A1) The quasi-static flat Rayleigh fading channel matrix *H* is perfectly known by the detector;

(A2) n is zero mean, circularly symmetric complex-valued Gaussian with $E(nn^H)=\sigma_n^2 I_{N_R}$;

(A3) x is zero mean and with $E(xx^H)=I_{N_T}$;

(A4) x *and* n are independent of each other.

III. Signal Detection Algorithm

Because MIMO-OFDM can be seen as a flat MIMO system, and there are several conventional MIMO detection algorithms. Some of them can be directly applied to MIMO-OFDM system.

A. Linear ZF Receiver

In a Zero-Forcing (ZF) receiver, the receiver vector y is multiplied by a filter matrix G.

$$G=(H^H H)^{-1}H^H \tag{3}$$

Get the estimation value of transmitted signals by

$$\tilde{x}=Gy \tag{4}$$

Where $\tilde{x}$ is the output of ZF.

B. QR Receiver

The QR decomposition of the channel matrix *H* was introduced in [5] as another method to decode MIMO-OFDM systems. The channel matrix *H* can be decomposed by $H=QR$ using the $N_R \times N_T$ unitary matrix *Q* and the $N_T \times N_T$ upper triangular matrix ***R***. So the model (1) can be amended as

$$y=QRx+n$$

$$Q^H y=Rx+Q^H n$$

$$\tilde{y}=Rx+\tilde{n} \tag{5}$$

Where $\tilde{y}=Q^H y$, $\tilde{n}=Q^H n$. The statistical properties of the noise term $Q^H n$ remain unchanged.

The (5) can be written as below

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ . \\ . \\ . \\ \tilde{y}_{N_T} \end{pmatrix} = \begin{pmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N_T} \\ 0 & r_{2,2} & \cdots & r_{2,N_T} \\ 0 & 0 & \cdots & r_{3,N_T} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ 0 & 0 & \cdots & r_{N_T N_T} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_{N_T} \end{pmatrix} + \begin{pmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ . \\ . \\ . \\ \tilde{n}_{N_T} \end{pmatrix} \tag{6}$$

The element N_T of vector $\tilde{y}$ is

$$\tilde{y}_{N_T}=r_{N_T,N_T}.x_{N_T}+\tilde{n}_{N_T} \tag{7}$$

Then, the decision statistic (7) can be used to estimate $\tilde{x}_{N_T}$, it is

$$\tilde{x}_{N_T}=Quant\left[\frac{\tilde{y}_{N_T}}{r_{N_T N_T}}\right] \tag{8}$$

Where *Quant* [] is the quantization operation.

If the decision of N_T layer is correct, thus its interference term can be subtracted from the N_T-1 layer signal $\tilde{y}_{N_T-1}$ when $\tilde{x}_{N_T-1}$ is estimated. The process of QR detecting can be described by using the following recursive algorithm,

$$\tilde{x}_i=\frac{1}{r_{i,i}}\left(\tilde{y}_i-\sum_{j=i+1}^{N_T} r_{i,j}\tilde{x}_j\right), i=N_T, N_T-1, N_T-2, ...1 \tag{9}$$

If the *ith* layer is detected successively and wrong, those wrong decisions will affect the subsequent decisions.

C. QRDM Receiver

QRDM algorithm is proposed for recovering the data streams in MIMO-OFDM systems [7]. *M* represents the number of branches that are kept in every layer. $M \in P=\{1,2,\ldots,S\}$, where *P* is the points set and *S* is the total points on the modulation constellation. For example, if 16QAM is used, then *M* can choose the value form 1 to 16 and *S* is equal to 16. In order to illustrate the process of QRDM detection, effective number of branches should be denoted as C_i, where $i=1,2,\ldots,N_T$. Algorithm:

Step1: Determine the number of branches should be calculated in ith layer;

(a) if i=1, then $C_1=S$;

(b) if i>1, then $C_i=SC_{i-1}$.

Step 2: Calculate the expansion weight metrics of each branch in *ith* layer by (10);

$$a_i(d)=\sum_{k=N_T+1-i}^{N_T}\left(\tilde{y}_k-\sum_{l=k}^{N_T} r_{k,l}x_l\right)^2, 1\le d\le C \tag{10}$$

Step 3: According to the value of $a_i(d)$, it need sort $a_i(d)$ from maximum to minimum;

(a) $i<N_T$, (I) if $C_i \leqslant M$, then i=i+1, go to step 1.

(II) if $C_i>M$, then *M* branches should be reserved and $C_i=M$, $i=i+1$; go to step 1.

(b) $i=N_T$, the algorithm is end and the points $\tilde{x}$ reserved in the smallest weight metrics of the branch is the output of QRDM algorithm.

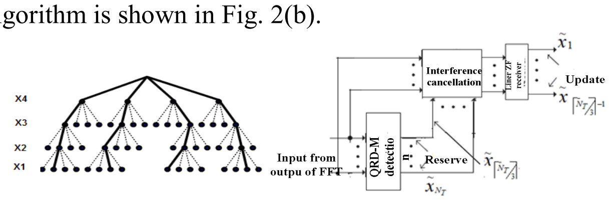
For example, when $N_T=N_R=4$, QPSK modulation, the tree structure of QRDM algorithm is shown in Fig.2 (a).

When M=1, the BER effective of QRDM receiver is similar to QR receiver. When M=S (the total points on the modulation constellation), the BER effective of QRD-M receiver is similar to Maximum likelihood (ML) receiver.

IV. A Hybrid QRDM and ZF Detection Algorithm

A. *Algorithm Development*

To the above MIMO-OFDM system model, a new detection scheme for MIMO channel matrix at every subcarrier is proposed. From [8], we can know when the first $N_T - \lceil N_T/3 \rceil + 1$ layers are detected by QRDM algorithm and the last $\lceil N_T/3 \rceil - 1$ layers are detected by ZF algorithm, the combined QRDM-ZF is reasonable, considering the compromise between the performance and the complexity of the proposed algorithm. The method consists of two parts. The first part is to apply the QRDM detection for obtaining the estimation of transmitting signals $[\tilde{x}_{\lceil N_T/3 \rceil}, \ldots, \tilde{x}_{N_T-1}, \tilde{x}_{N_T}]$, where $\lceil N_T/3 \rceil$ represent the minimum integer number that is bigger than $N_T/3$ or equal to $N_T/3$. The second part is an operation like conventional linear zero-force algorithm. The structure of the proposed hybrid QRDM and ZF detection algorithm is shown in Fig. 2(b).



(a) MIMO-OFDM system QRDM（M=4）tree structure with QPSK modulation

(b) The structure of the hybrid QRDM and ZF detection

Figure 2. MIMO-OFDM system QRDM（M=4）tree structure with QPSK modulation and the structure of the hybrid QRDM and ZF detection

The hybrid QRDM and ZF detection algorithm based on MIMO-OFDM system scheme is described as follows:

Step1: Denote set $A=\{1,2,\ldots,N_T\}$, set $B=\emptyset$.

For i=1 to N

$$k = \arg\min_{k \in A} \left\| H_{(k)} \right\|^2 ;$$

$$Hnew_{(i)} = H_{(k)},\ A = A \backslash k, B = B \cup k;$$

End

Where $H_{(k)}$ represents the kth column of the matrix H and $Hnew$ is the channel matrix after sorting and its columns sorts as the gain of $H_{(k)}$ from minimum to maximum.

Step2: The conventional QRDM detection algorithm is firstly adopted. However, the algorithm should be stopped at $i = N_T - \lceil N_T/3 \rceil + 1$.

Step3: Reserved the signals of the $N_T - \lceil N_T/3 \rceil + 1$ detected layers and the signals can be denoted as $x^* = [\tilde{x}_{\lceil N_T/3 \rceil}, \ldots, \tilde{x}_{N_T-1}, \tilde{x}_{N_T}]^T$. It may cancel the interference of reserved signal to the other layers. Then a new MIMO-OFDM system with $(\lceil N_T/3 \rceil - 1, N_R)$ antennas is obtained. For the new MIMO-OFDM system the signal relation between input and output is

$$y_{Update} = y - H^* x^* \qquad (11)$$

Where $H^* = [Hnew_{\lceil N_T/3 \rceil}, \ldots, Hnew_{N_T}]$.

The channel matrix is update by

$$H_{Update} = [Hnew_1, \ldots, Hnew_{\lceil N_T/3 \rceil - 1}]$$

And the filter matrix G_{Update}.

$$G_{Update} = (H_{Update}{}^H H_{Update})^{-1} H_{Update}{}^H \qquad (12)$$

Step4: Use the conventional linear ZF detection. The other $\lceil N_T/3 \rceil - 1$ signals are estimated by $x_{others} = G_{Update} y_{Update}$ $= [\tilde{x}_1, \ldots, \tilde{x}_{\lceil N_T/3 \rceil - 1}]^T$ and all signals $\tilde{x} = [x_{others}{}^T, x^{*T}]^T$ are obtained.

Step5: Finally, sorting $\tilde{x}$ corresponding to the order in set B and the output is the result of the hybrid QRDM-ZF algorithm detection.

B. *Algorithm Analysis*

In the first part of the detection, the channel matrix H has been sorted as the column norm from minimum to maximum. The first detection signal is corresponding to the maximum column norm representing the maximum signal-to-noise (SNR) layer. As it's known, the bigger the SNR is, the more reliable of the detection signal is. By this way, the performance of the first detected signals has enhanced greatly. However, when it comes to the last few layers, the SNR is small. In order to lower the complexity of the algorithm, ZF receiver rather than QRDM receiver is used in the second part of the detection. Due to the performance of the first part $N_T - \lceil N_T/3 \rceil + 1$ detected layers is the best in all layers, the performance of ZF detection will not influence the overall performance too much. Furthermore, when the $N_T - \lceil N_T/3 \rceil + 1$ signals are reserved, its interference to other layers may be cancelled. The simulation results of this determination method will be given in Section V.

V. Simulation Results

In this section, computer simulations are presented to evaluate the performance of the proposed dynamic hybrid QRDM-ZF algorithm. The receiver detection is based on five paths Rayleigh fading environment without channel coding, assuming perfect channel estimation. For comparison purpose, ZF and QRDM receivers are also presented. In the simulation, the FFT size is 64, and the cyclic prefix length L is 16. The antenna configuration consists of 4 transmitter antennas, and 4 receiver antennas. The QPSK and 16-QAM modulation are used. Fig. 3 shows the comparison performance of three algorithms with different M (M=1, 2, 3, 4) by QPSK modulation and Fig. 4 explains the performance of those algorithms with M (M=8, 12, 16) by 16QAM modulation.

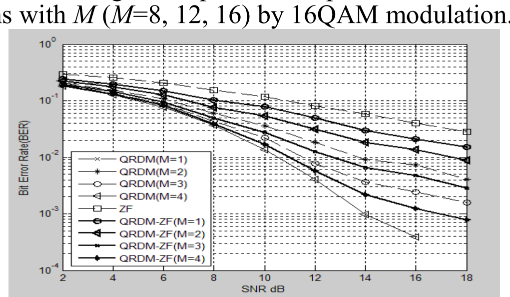


Figure.3 BER performance of ZF, QRDM and QRDM-ZF with M=1,2, 3, 4 by QPSK modulation in 4×4 MIMO-OFDM system without channel coding.

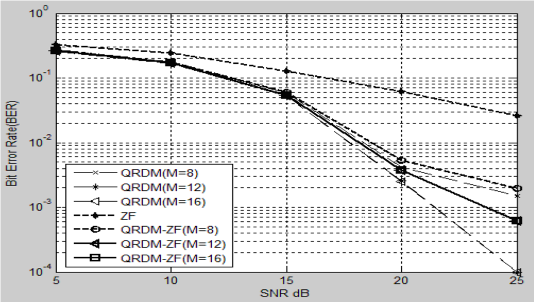


Figure.4 BER performance of ZF, QRDM and QRDM-ZF with M=8,12,16 by 16QAM modulation in 4×4 MIMO-OFDM system without channel coding.

A. The Performance of Hybrid QRDM-ZF Algorithm Analysis

From Fig.3 and Fig.4, it can be concluded that the performance of QRDM-ZF algorithm is better than that of ZF detection. However, even thought QRDM-ZF detection algorithm has lower complexity than QRDM algorithm, its performance is not much worse than QRDM algorithm. Compared with the conventional QRDM algorithm, the performance is about 2dB degradation at BER=10^{-3} with 16-QAM modulation and 2.1dB degradation at BER=10^{-3} with QPSK modulation. The reason is that the channel matrix H has been sorted as the column norm from the minimum to the maximum. The performance of the first detected signals has enhanced greatly and it's reasonable to use ZF detection to the last few layers with small SNR.

B. Complexity Analysis

The branches that QRDM algorithm should search are defined as (13).

$$L = S + \sum_{i=1}^{N_T - 1} M S \qquad (13)$$

Where L is the total branches that should be searched, S and M are defined above in the QRDM receiver.

The effective branches of QRDM denote as L_{QRDM} and the effective branches of QRDM-ZF denote as $L_{QRDM\text{-}ZF}$. L_{QRDM} and $L_{QRDM\text{-}ZF}$ are defined in (14) and (15).

$$L_{QRDM} = S(MN_T - M + 1) \qquad (14)$$

$$L_{QRDM-ZF} = S(MN_T - M\lceil N_T / 3 \rceil + 1) \qquad (15)$$

Due to the less complexity of linear ZF detection, the complexity between QRDM and hybrid QRDM-ZF can be compared by effective searching branches. For 4 transmitter antennas and 4 receiver antennas, with QPSK and 16QAM modulation, the effective branches base on different M are shown in Table I and Table II.

TABLE I.
DIFFERENT SEARCH BRANCHES OF QRDM AND QRDM-ZF BASED ON QPSK, N_R=N_T=4

Value of M	L_{QRDM}	$L_{QRDM\text{-}ZF}$	Reduced complexity	Average reduced complexity
1	16	12	25%	29.41%
2	28	20	28.57%	
3	40	28	30%	
4	52	36	30.77%	

TABLE II.
DIFFERENT SEARCH BRANCHES OF QRDM AND QRDM-ZF BASED ON 16QAM, N_R=N_T=4

The value of M	L_{QRDM}	$L_{QRDM\text{-}ZF}$	Reduced complexity	Average reduced complexity
1	64	48	25%	32.08%
2	112	80	28.57%	
3	160	112	30%	
4	208	144	30.77%	
5	256	176	31.25%	
6	304	208	31.58%	
7	352	240	31.82%	
8	400	272	32%	
9	448	304	32.14%	
10	496	336	32.26%	
11	544	368	32.35%	
12	592	400	32.43%	
13	640	432	32.5%	
14	688	464	32.56%	
15	736	496	32.61%	
16	784	528	32.65%	

Form table I and table II, it can be concluded that the proposed scheme compared with QRDM algorithm is quite effective with a reduced complexity. In a 4×4 MIMO-OFDM system by QPSK modulation, compared with QRDM algorithm, the proposed technique has decreased the complexity by 29.41% on average. In the same systems by 16-QAM modulation, the hybrid QRDM-ZF algorithm has decreased the complexity by 32.08% on average.

VI. CONCLUSIONS

In this paper, a new efficiency algorithm for MIMO-OFDM system was proposed. It enhances the detection performance by using the QRD-M and reduces the complexity by applying the conventional ZF detection algorithm. Simulation results show that if the complexity of the detection algorithm is reduced, the detection performance will decrease. In fact, a signal detection algorithm with a low complexity and high BER performance is a troublesome subject. The traditional QRD-M is difficult because of its high complexity. Therefore, the proposed hybrid QRDM-ZF detection can be used for a MIMO-OFDM receiver requiring not very high performance but needing very low complexity.

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